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A new statistical damage theory

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The shortcomings of continuum damage mechanics (CDM) are discussed and non-equilibrium statistical physics is used to establish a new statistical theory of inhomogeneous damage. The initiation and growth of microscopically damaged regions (cracks, voids, etc.) is regarded as the elementary process of damage to the material structure and the accumulation damage, i.e. damage variable, is universally defined as the failure probability of the material due to the initiation and growth of the microscopically damaged regions. From the statistical evolution equation of damaged regions, and the minimum strength principle, a partial differential equation, which universally describes the evolution of damage parameter, is found. Not only can this equation characterize the kinetic process of damage evolution, but it can also establish the relationships between the microbehaviour of damage (the initiation and growth of the microscopically damaged regions and the statistical consequences of damage) and the degradation of material properties. Finally, as an example, the newly developed theory is applied to study the time-dependent fracture of Al_2O_3 ceramic. The effects of structural inhomogeneity on mechanical properties of the material is discussed.

1. Introduction

The concept of damage has been widely accepted in describing the evolution of material structure and the degradation of mechanical properties of materials subjected to externally applied stress and environment. But the mathematical representation and the physical meaning of the damage variable are still not very certain. According to continuum damage mechanics (CDM), the general definition of a damage variable, w, is [1]

$$0 \leqslant w \leqslant 1 \tag{1a}$$

where

$$w = \begin{cases} 0 & \text{for a virgin material} \\ 1 & \text{for a failed material} \end{cases} \tag{1b}$$

Apart from this, there is no rigorous form of definition for damage variable. For this reason there are many forms of definition for the damage variable given for different models. Some are defined by the effective cross-section or the effective volume of materials [1, 2], some by defect density (cracks, voids, etc.) [3], some by the density of strain energy [4], but none is universally accepted. In the discussion about the physical meaning of damage, it is suggested that the damage variable can be appropriately interpreted as the impairment of the stress-transmitting capacity of a material structure as a result of the presence of defects [3]. But the impairment is neither conveniently evaluated from theory nor easily measured from experiments. In order to study the time-dependent evolution of material damage, Kachanov also proposed the kinetic equation of damage phenomenologically [1]

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{C\sigma^p}{(1-w)^q} \tag{2}$$

where C, p, q are all material constants to be determined by experiments.

Investigating the theory of CDM we can find that it has two obvious flaws, even though it is quite successful in studying the damage of various materials in different situations. One is that the definition of damage variable is not unique and the relations between the different definitions are not clear. This caused some confusion in the understanding of damage and spawned a host of substantially different methods dealing with the same phenomenon. The other is that the damage parameter and its kinetic equation do not correlate with the micromechanism of structure damage, especially the localized damage, so that the connections between the damage behaviour of microstructure and the degradation of macroscopic properties of materials are not clear either. Consequently, it appears necessary to give the damage parameter a mathematically, and physically in particular, acceptable definition and to find theoretically a micromechanism-dependent kinetic equation of damage.

Actually, in principle, the damage to a material caused by applied stress originated from the deteriorating evolution of microstructure in the form of cracks, void or other forms of defects, while the degradation of different mechanical properties of the material, the Young's modulus for example, is only the representation of microstructure evolution in different aspects. But, on the contrary, some representations in a specified aspect are not eligible to characterize the whole behaviour of material damage. Hence the abovementioned different definitions of damage variable are all not theoretically appropriate, even though they are usually very simple for practical use. The evolution of microstructure, for example cracking, is usually inhomogeneous and localized, it varies randomly for different time and locations due to the inhomogeneous microstructure, hence it causes the microscopic damage to the material structure, while the damage parameter expressed by Equation 1 is the accumulation damage or macroscopic damage of the material. It presents the statistical behaviour of microscopic damage. Therefore, an ideal definition of the damage parameter should, on the one hand, correlate with the mechanism of microscopic damage and, on the other hand, has statistical meaning. The damage parameter so defined can naturally realize the connections between the micromechanism of structure evolution and mechanical properties of the material. In this paper, from the viewpoint of stochastic evolution of microscopic damage, we use the theory of non-equilibrium statistical physics, to establish a new statistical damage theory and derive a partial differential equation for the damage parameter from the statistical evolution equation of microscopically damaged regions and the minimum strength principle, which can universally describe the evolution of the damage parameter with time due to the initiation and growth of damaged regions in the material. Finally, we give an example for the application of the newly developed theory.

2. Evolution of failure probability

The process of failure of a material is the process in which many microscopically damaged regions (cracks, voids, etc.) initiate and grow continuously in the material under the action of applied stress and then the most critical one of them propagates unstably, causing the catastrophic failure of the material. Because the microstructure of a material is usually inhomogeneous, the growth rate of the damage regions is stochastic. For convenience of handling, the microstructure of the material is considered as the average structure background superimposed by the inhomogeneous structure fluctuation due to all kinds of inhomogeneity. Then the growth rate of the size of a damaged region, a, is expressed as [5, 6]

$$\frac{\mathrm{d}a}{\mathrm{d}t} = M(a) + \beta(a) f(t) \tag{3}$$

where $M(a)$ is the migration growth rate, which is determined by the average structure background of the material and the applied stress, $f(t)$ is the fluctuation function, which is related to the stochastic fluctuation of microstructure and the applied stress, $\beta(a)$ is a fluctuation magnifying function. $f(t)$ is assumed to be the white noise, so

$$\begin{cases} \langle f(t) \rangle = 0 \\ \langle f(t) f(t') \rangle = Q\delta(t - t') \end{cases} \tag{4}$$

where Q is the fluctuation coefficient, which characterizes the effect of microstructure fluctuation on the growth of the damaged regions. The Fokker–Planck equation in equivalence to Equation 3 [7], which describes the evolution of microscopically damaged regions, is

$$\frac{\partial p(a, t)}{\partial t} = -\frac{\partial}{\partial a}\left\{\left[M(a) + \frac{Q}{2}\beta(a)\frac{\partial \beta(a)}{\partial a}\right] p(a, t)\right\} + \frac{Q}{2}\frac{\partial^2}{\partial a^2}[\beta^2(a) p(a, t)] \tag{5}$$

where $p(a, t)\,\mathrm{d}a$ is the probability that a damaged region grows to a size between a and $a + \mathrm{d}a$ at time t. When a large number of microscopically damaged regions is considered the failure probability of the material can be obtained from the minimum strength principle as follows [5, 6]

$$F(\sigma, t) = 1 - \left[1 - \int_{a(\sigma)}^{\infty} p(a, t)\,\mathrm{d}a\right]^{\rho \bar{V}} \tag{6}$$

where $a(\sigma)$ is the critical size of the damaged regions, which is a monotonically decreasing function of applied stress σ, ρ is the density of the damaged regions and the V is the material volume. Differentiating Equation 6 with respect to t, we have

$$\frac{\partial F}{\partial t} = \rho V (1 - F)^{1 - \frac{1}{\rho V}} \int_{a(\sigma)}^{\infty} \frac{\partial p(c, t)}{\partial t}\,\mathrm{d}a - (1 - F)\ln(1 - F)\frac{\mathrm{d}\ln \rho}{\mathrm{d}t} \tag{7}$$

Substituting Equation 5 into the above equation, we obtain

$$\frac{\partial F}{\partial t} = \rho V (1 - F)^{1 - \frac{1}{\rho V}} \left\{\left[M(a) + \frac{Q}{2}\beta(a)\frac{\partial \beta(a)}{\partial a}\right] \times p(a, t) - \frac{Q}{2}\frac{\partial}{\partial a}[\beta^2(a) p(a, t)]\right\}_{a(\sigma)} - \frac{\mathrm{d}\ln \rho}{\mathrm{d}t} \times (1 - F)\ln(1 - F) \tag{8}$$

Then differentiating Equation 6 with respect to a once and twice, we find

$$\frac{\partial F}{\partial a} = \rho V (1 - F)^{1 - \frac{1}{\rho V}} [-p(a, t)]_{a(\sigma)} \tag{9}$$

$$\frac{\partial^2 F}{\partial a^2} = \left[\rho V(\rho V - 1)(1 - F)^{1 - \frac{2}{\rho V}} p^2(a, t) - \rho V (1 - F)^{1 - \frac{1}{\rho V}} \frac{\partial p(a, t)}{\partial a}\right]_{a(\sigma)} \tag{10}$$

Inserting Equations 9 and 10 into Equation 8 yields the evolution equation of failure probability

$$\frac{\partial F}{\partial t} + \left\{\left[M(a) - \frac{Q}{2}\beta(a)\frac{\partial\beta(a)}{\partial a}\right]\frac{\partial F}{\partial a} - \frac{Q}{2}\beta^2(a)\frac{\partial^2 F}{\partial a^2} + \frac{Q}{2}\frac{\beta^2(a)}{1-F}\left(\frac{\partial F}{\partial a}\right)^2\right\}_{a(\sigma)} = -\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}(1-F)\ln(1-F) \quad (11)$$

From this equation it can be seen that the change rate of failure probability of a material with time is determined by the initiation, growth and fluctuation of microscopically damaged regions, which are characterized by $\mathrm{d}\rho/\mathrm{d}$, $M(a)$, $\beta(a)$ and Q, respectively. If the non-linear term in Equation 10, $p^2(a, t)$, is considered small, Equation 11 reduces to the first Kolmogorov equation as follows

$$\frac{\partial F}{\partial t} + \left\{\left[M(a) - \frac{Q}{2}\beta(a)\frac{\partial\beta(a)}{\partial a}\right]\frac{\partial F}{\partial a} - \frac{Q}{2}\beta^2(a) \times \frac{\partial^2 F}{\partial a^2}\right\}_{a(\sigma)} = -\frac{\mathrm{d}\ln\rho}{\mathrm{d}t}(1-F)\ln(1-F) \quad (12)$$

It should be noted that the failure probability can be directly obtained by solving Equation 12 when $\mathrm{d}\rho/\mathrm{d}$, $M(a)$, $\beta(a)$ and Q are known and there is no need to solve the probability distribution function of damaged regions $p(a, t)$ as we did in previous work [5, 6].

3. Evolution equation of damage

Failure probability $F(\sigma, t)$ determined from Equation 12 not only correlates with the mechanism of microstructure evolution but also has statistical meaning, meanwhile it coincides with the general definition of damage variable (Equation 1). Therefore, we defined the failure probability as the damage variable, w[5, 6, 8, 9], that is

$$w[a(\sigma), t] \equiv F(\sigma, t) \quad (13)$$

This definition is universal, for it is applicable to different processes of damage to various materials as long as the initiation and growth behaviour of microscopically damaged regions are known. Furthermore, it also characterizes the physical meaning of damage interpreted as the impairment of the stress-transmitting capacity of material structure given by Krajcinovic. Because the impairment of the stress-transmitting capacity is caused by the initiation and growth of microscopically damaged regions under the action of applied stress, then the failure probability can statistically present the extent of the impairment. However, this definition is neither too abstract to evaluate theoretically nor too concrete to characterize the damage of material microstructure as a whole. Therefore, this definition of damage variable is physically acceptable. Substituting Euation 13 into Equation 12, we obtain the evolution equation of damage as follows

$$\frac{\partial w}{\partial t} + \left\{\left[M(a) - \frac{Q}{2}\beta(a)\frac{\partial\beta(a)}{\partial a}\right]\frac{\partial w}{\partial a} - \frac{Q}{2}\beta^2(a)\frac{\partial^2 w}{\partial a^2}\right\}_{a(\sigma)} = -\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}(1-w)\ln(1-w) \quad (14)$$

The damage parameter can be directly solved from the above equation in combination with some initial and boundary conditions. When the fluctuation of the growth rate of the damaged regions is negligibly small, the above equation reduces to a simpler form

$$\frac{\partial w}{\partial t} + \left[M(a)\frac{\partial w}{\partial a}\right]_{a(\sigma)} = -\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}(1-w)\ln(1-w) \quad (15)$$

which corresponds to the damage in a uniform material. If the total number of damaged regions is a constant, $\mathrm{d}\rho/\mathrm{d}t = 0$, above equation reduces to an even simpler form

$$\frac{\partial w}{\partial t} + \left[M(a)\frac{\partial w}{\partial a}\right]_{a(\sigma)} = 0 \quad (16)$$

For the case where many different mechanisms of damage to material microstructure exist, the evolution equation of damage can also be established in the same way, but its form will be more complex. The discussion about the evolution equation of damage for many mechanisms will be given elsewhere.

4. Example of application

In order to show the performance of our theory, we take the time-dependent fracture of brittle materials due to the slow growth of pre-existing cracks under uniaxial tension as an example. First, we derive the time-dependent damage parameter and the average lifetime for both homogeneous and inhomogeneous materials. Then we apply above theoretical results to Al_2O_3 ceramic and see how the structural inhomogeneity affects the mechanical properties of the material.

4.1. Homogeneous materials

In this case, the damage mechanism is the slow growth of the pre-existing cracks. The slow crack growth rate is usually expressed as [10]

$$\frac{\mathrm{d}a}{\mathrm{d}t} = M(a) = A(\sigma Y)^n a^{\frac{n}{2}} \quad (17)$$

where A, n are two material constants, Y is the geometric factor. The critical condition for unstable propagation of cracks is

$$a(\sigma) = \left(\frac{K_{\mathrm{Ic}}}{Y\sigma}\right)^2 \quad (18)$$

where K_{Ic} is the fracture toughness of the material. Consequently, according to Equation 16, the evolution equation of damage parameter should be

$$\frac{\partial w}{\partial t} + \left[A(\sigma Y)^n a^{\frac{n}{2}} \frac{\partial w}{\partial a} \right]_{a=(K_{Ic}/Y\sigma)^2} = 0 \quad (19)$$

Here the initial condition is assumed to be the two-parameter Weibull function [9]

$$w[a(\sigma), t = 0] = 1 - \exp\left[-\rho V \left(\frac{\sigma}{\sigma_0} \right)^m \right] \quad (20)$$

where m is the Weibull modulus and σ_0 is the stress corresponding to the smallest crack size, a_0. Solving Equations 19 and 20 we obtain the damage parameter as

$$w(\sigma, t) = 1 - \exp\left\{ -\rho V \left(\frac{\sigma}{\sigma_0} \right)^m \times \left[1 + \left(\frac{n-2}{2} \right) A(\sigma Y)^2 K_{Ic}^{n-2} t \right]^{\frac{m}{n-2}} \right. \quad (21)$$

which is identical to the result obtained in another way [11]. With $w(\sigma, t)$ we can calculate the average lifetime of a material, t_f, corresponding to a specified stress, σ.

$$t_f = \int_0^\infty t \frac{\partial w}{\partial t} \mathrm{d}t = \int_0^\infty (1 - w) \mathrm{d}t \quad (22)$$

Substituting Equation 21 into the above equation and calculating, we obtain

$$t_f = \frac{\sigma_0^{n-2}}{\{[(n-2)/2] A Y^2 K_{Ic}^{n-2} \sigma^n\}} \frac{\Gamma[1 + (n-2)/m]}{(\rho V)^{(n-2)/m}} \quad (23)$$

In the above calculation, we have used the approximation that $A\sigma^2 K_{Ic}^{n-2} t \gg 1$. With Equation 23, the damage parameter can be rewritten in a simpler form

$$w(\sigma, t) = 1 - \exp\left\{ -\left[\Gamma\left(1 + \frac{n-2}{m} \right) \frac{t}{t_f} \right]^{\frac{m}{(n-2)}} \right\} \quad (24)$$

It is also a Weibull function with the shape and scale parameters fully determined by the applied stress and the material characteristics.

4.2. Inhomogeneous materials

For the case of inhomogeneous materials, the slow crack growth rate must be expressed by a stochastic equation such as Equation 3; then we have [5, 6]

$$\frac{\mathrm{d}a}{\mathrm{d}t} = A(\sigma Y)^n a^{n/2} + a^{n/2} f(t) \quad (25)$$

From Equation 14 the corresponding evolution equation of damage should be

$$\frac{\partial w}{\partial t} + \left\{ \left[A(\sigma Y)^n a^{n/2} - \frac{Q}{4} a^{n-1} \right] \frac{\partial w}{\partial a} - \frac{Q}{2} a^n \frac{\partial^2 w}{\partial a^2} \right\}_{a=(K_{Ic}/Y\sigma)^2} = 0 \quad (26)$$

in which the fluctuation coefficient, Q, is defined by Equation 4. Solving Equations 26 and 20, we find the damage parameter to be

$$w(\sigma, t) = 1 - \frac{N}{\pi^{1/2}} \int_{-(A_1^2 t/2Q)^{1/2}}^{\xi_0} \exp\left(-\rho V \left(\frac{\sigma}{\sigma_0} \right)^m \times \left\{ 1 + A_1 (\sigma Y)^2 K_{Ic}^{n-2} t \left[1 + \left(\frac{2Q}{A_1^2 t} \right)^{1/2} \times \xi \right] \right\}^{m/(n-2)} \right) \exp(-\xi^2) \mathrm{d}\xi \quad (27)$$

where

$$A_1 = A(\sigma Y)^n \quad (28)$$

$$\xi_0 = \left[\frac{2}{n-2} (a_0^{-\frac{n}{2}+1} - a^{-\frac{n}{2}+1}) - A_1 t \right] \Big/ (2Qt)^{1/2} \quad (29)$$

and N is a normlization factor.

$$N^{-1} = \frac{1}{\pi^{1/2}} \int_{-(A_1^2 t/2Q)^{1/2}}^{\xi_0} \exp(-\xi^2) \mathrm{d}\xi \quad (30)$$

Although the integration in Equation 27 cannot be calculated analytically, we are able to see some features of $w(\sigma, t)$. When t tends to zero, $w(\sigma, t)$ reduces to the initial damage (Equation 20). On the contrary, when t tends to infinity, we have $w(\sigma, t \to \infty) = 1$, which means that if the duration of the load is very large, failure of the material will definitely occur. Furthermore, because $a(\sigma)$ is a monotonically decreasing function of applied stress, σ, from Equation 27 we can also find that $w(\sigma \to 0, t) = 0$ and $w(\sigma \to \infty, t) = 1$. This indicates that the failure of the material does not occur without the action of applied stress and the failure will definitely occur when the applied stress is very large. Similarly, the average lifetime of a material can be obtained by combining Equations 22 and 27

$$t_f = \int_0^\infty N \int_{-(A_1^2 t/2Q)^{1/2}}^{\xi_0} \exp\left(-\rho V \left(\frac{\sigma}{\sigma_0} \right)^m \times \left\{ 1 + \frac{n-2}{n} A(\sigma Y)^2 t K_{Ic}^{n-2} \times \left[1 + \left(\frac{2Q}{A_1^2 t} \right)^{1/2} \xi \right] \right\}^{\frac{m}{n-2}} - \xi^2 \right) \mathrm{d}\xi \, \mathrm{d}t \quad (31)$$

which can only be calculated numerically. If Q tends to zero, Equations 22 and 31 will reduce to the damage parameter and the average lifetime corresponding to homogeneous materials, Equations 21 and 23.

4.3. Calculation

Here we apply the above results to 96% Al_2O_3 ceramic with average grain size of 32 μm under eccentric loading in which the cracks grow due to the tensile stress. The slow crack growth rate is expressed as a power function of the ratio of stress intensity factor over the fracture toughness [12].

$$\frac{da}{dt} = B\left(\frac{K_a}{K_{Ic}}\right)^n \tag{32}$$

In comparison with Equation 17 it is easily found that

$$A = \frac{B}{K_{Ic}^n} \tag{33}$$

With Equations 33, 21 and 27 we can calculate the failure probability function of the material with homogeneous and inhomogeneous structures, respectively. The material parameters used for calculation are listed in Table I. The failure probability of the material under different applied stresses for the case where $Q = 0.0$ is shown in Fig. 1. We can see that the failure probability increases obviously with the increase of applied stress. When the structural inhomogeneity is considered the failure probability will deviate from that for homogeneous structure, depending on the value of the fluctuation coefficient. Figs 2–4 show the variations of failure probability corresponding to different values of fluctuation coefficient. The solid, dashed and long dashed lines correspond, respectively, to a homogeneous structure and a structure with small inhomogeneity characterized by Q_1, and a very inhomogeneous structure characterized by Q_2. The parameters used for calculation are listed in Table I

The relation between the fluctuation coefficient and the structural inhomogeneity has been discussed elsewhere [13]. From the figures it is seen that the failure probability decreases with the increase of fluctuation

TABLE I Material characteristics of 96 % Al_2O_3 ceramic

Weibull modulus	$M = 10$
Flaw density	$\rho = 10^6\ m^{-3}$
Volume of specimen	$V = 10^{-3}\ m^3$
Initial crack size	$a_0 = 40$ μm
Parameters in crack growth law	$n = 31$
	$B = 6.74\ m\,s^{-1}$
	$Y = 1.27$
Fracture toughness	$K_{Ic} = 5.3$ MPa
Fluctuation coefficient	$Q_1 = 3.34 \times 10^{34}\ \sigma^n$
	$Q_2 = 8.35 \times 10^{35}\ \sigma^n$

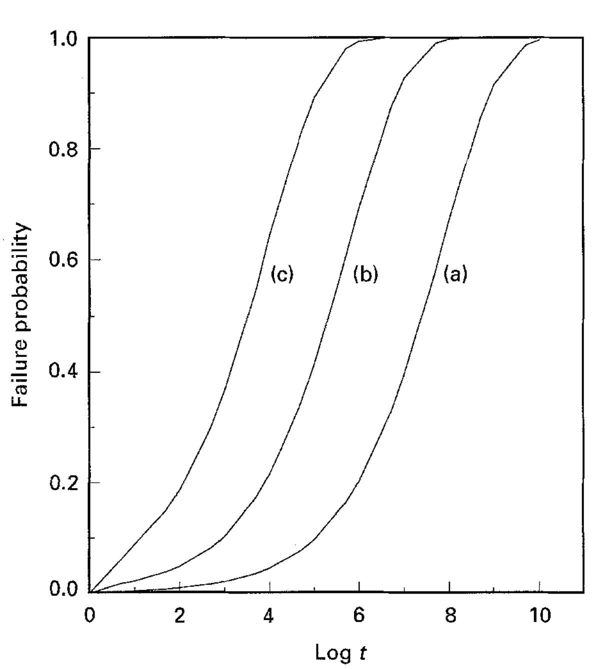

Figure 1 Failure probability for different applied stresses. (a) $\sigma = 120$ MPa, (b) $\sigma = 140$ MPa, (c) $\sigma = 160$ MPa.

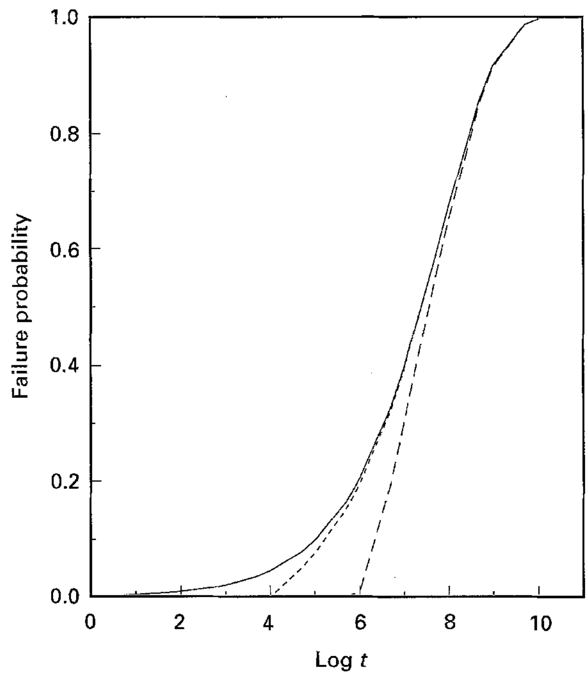

Figure 2 Failure probability under 120 MPa for different values of fluctuation coefficient. (——) $Q = 0.0$, (---) Q_1, (— — —) Q_2.

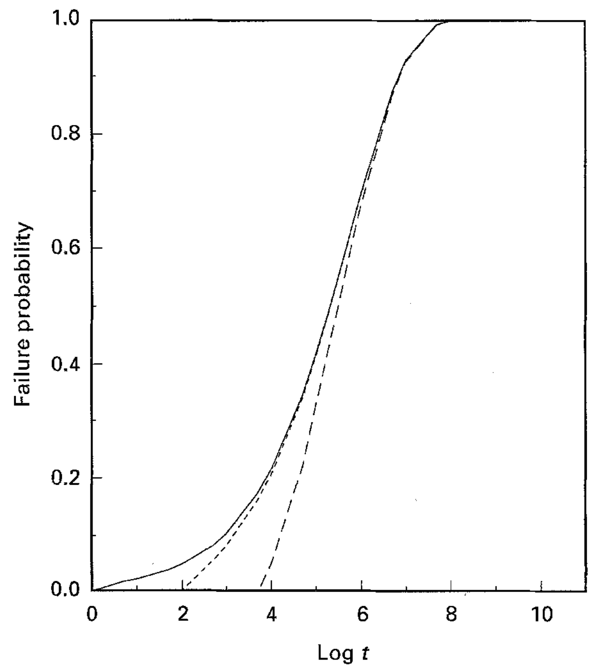

Figure 3 Failure probability under 140 MPa for different values of fluctuation coefficient. (——) $Q = 0.0$, (---) Q_1, (— — —) Q_2.

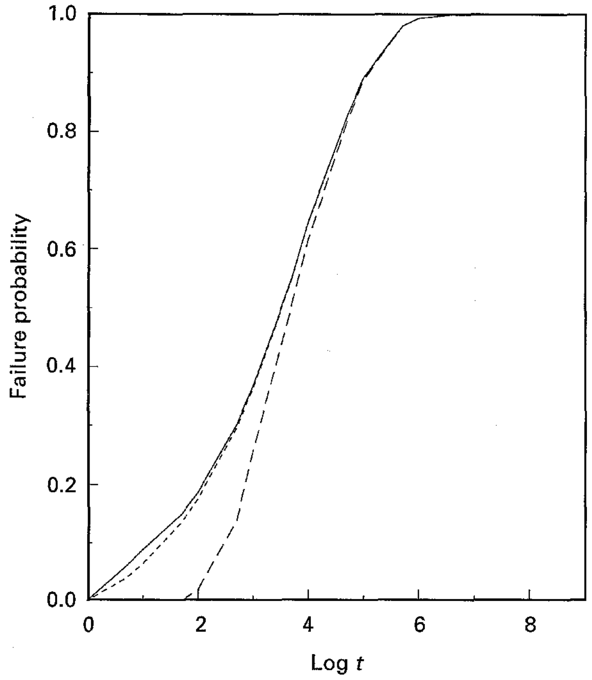

Figure 4 Failure probability under 160 MPa for different values of fluctuation coefficient. (——) $Q = 0.0$, (– – –) Q_1, (– – – –) Q_2.

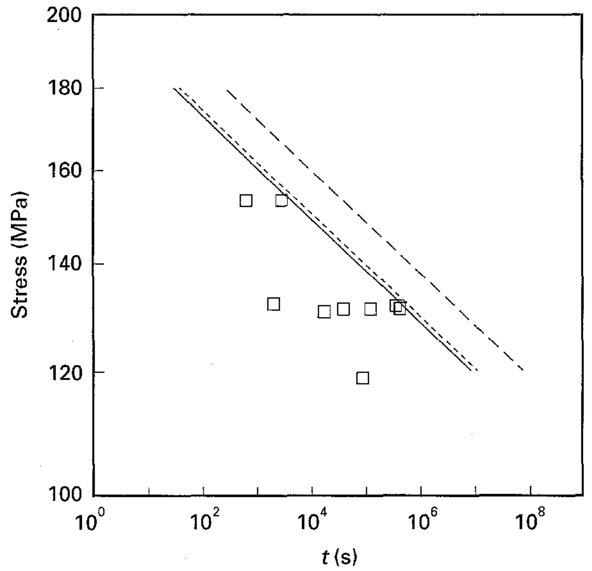

Figure 5 The $\log\sigma$–$\log t$ elation for 96% Al_2O_3 corresponding to different values of fluctuation coefficient. (——) $Q = 0.0$, (– – –) Q_1, (– – – –) Q_2, (□) Experimental data.

coefficient, but this effect becomes less obvious when the applied stress and the duration of stress action are very large. Physically, this means that an inhomogeneous structure is more reliable than a homogeneous structure. Although it is widely accepted that an inhomogeneous structure is more resistant to crack growth and some models have been proposed to explain this phenomenon [14], how the structural inhomogeneity of material affects the global response of the material is still unclear. The theory developed in this paper provides a quantitative explanation to this problem. The calculated stress–lifetime curves corresponding to different values of the fluctuation coefficient according to Equations 23 and 32 and the comparison with experimental data [12] are shown in Fig. 5. The linear relation between $\log\sigma$ and $\log t$ can be found. The figure indicates that an inhomogeneous material is of longer lifetime than a homogeneous material. Because no systematic experimental data on the effects of structural inhomogeneity on mechanical properties of materials have been found, this result can only serve as a prediction to be verified experimentally.

5. Conclusion

The concept of damage has been widely used for decades, but its physical meaning has always been ambiguous to some extent. Continuum damage mechanics, which is successfully used to study the degradation of material properties, also has some flaws in its theory so that it cannot satisfactorily describe the inhomogeneous damage and establish the relationship between the micromechanism of material damage and the mechanical properties of materials. Therefore, theoretical improvement and development are needed.

In this paper, the growth of microscopically damaged regions has been considered as the elementary process of damage to a material structure and the non-equilibrium statistical method was used to establish the statistical damage theory. On the basis of the universal definition of damage parameter, which is defined as the failure probability of a material, a new partial differential equation or the evolution equation of damage has been developed to describe the evolution of macroscopic damage or accumulation damage of the material due to the initiation and growth of microscopically damaged regions (cracks, voids, etc.) under the action of applied stress. The evolution equation of damage is applicable to the study of the damage evolution of various materials for different cases, for instance, fatigue fracture of materials under simple tension or multi-axial load, as long as the micromechanisms of initiation, growth and propagation of microscopically damaged regions are known. As an example of the application of our newly developed theory, the time-dependent fracture of brittle materials with a pre-existing flaw distribution has been discussed. The calculated results for 96% Al_2O_3 ceramic under eccentric loading, show that an inhomogeneous material is more reliable than a homogeneous one, especially when the applied stress is low and in the early stage of crack growth, and the lifetime of a material increases with increase of its structural inhomogeneity. As compared with the theory of CDM, our theory has the following advantages.

1. The statistical damage theory is based on the kinetic foundation of microstructure evolution and realizes the connection between the micromechanism of structural damage and the global response of materials naturally.

2. The evolution equation of damage is much more substantial and phenomenologically effective than the kinetic equation of damage proposed by Kachanov in describing material damage.

The statistical damage theory established in this paper can be further developed to describe the damage evolution due to different parallel mechanisms. This discussion will be presented separately.

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